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Stochastic factorizations, sandwiched simplices and the topology of the space of explanations. (English summary)

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This paper applies ideas from Morse theory to a problem in statistics. Specifically, the authors consider pairs of random variables X and Y which take on only finitely many values. The goal is to understand the possibilities for a third unobserved random variable Z such that X and Y are conditionally independent given Z. This is equivalent to understanding factorizations of the stochastic $m \times n$ matrix $V = (v_{i,j})$, where $v_{i,j} = P(Y = y_j | X = x_i)$, as a product $V = UW$, where U is an $m \times r$ stochastic matrix, W is an $r \times n$ stochastic matrix and $r = \mathrm{rank}(V)$. Let $\tilde{SF}_r(V)$ be the space of all such factorizations and $SF_r(V)$ the quotient modulo the symmetric group S_r acting on columns of U and rows of W, i.e., modulo aliasing of Z. One result of this paper is a characterization of $\tilde{SF}_r(V)$ in terms of the space of ordered $(r-1)$-simplices lying between two polyhedra in $\mathbb{R}^{r-1}$. As a consequence, it is shown that, for $r = 2$, $SF_2(V)$ is homeomorphic to an interval. For $r = 3$, $SF_3(V)$ may be empty, a homotopy circle or homotopy equivalent to k points for $0 \leq k \leq m + n$. If columns of V are "nearly parallel", then $\tilde{SF}_r(V)$ is proven to be homotopy equivalent to the space of $(r-1)$-simplices with vertices on the $(r-2)$-sphere S^{r-2}. For higher ranks, Morse theory is developed to help understand the homology of the space of factorizations.

Reviewed by *Richard Stong*